Int. J. Heat Mass Transfer. Vol. 36, No. 3, pp. 735–741, 1993
Printed in Great Britain

0017-9310/93 $6.00 + 0.00

On the enhancement in heat transfer through the use of slurries

M. S. PHANIKUMAR
CSIR Centre for Mathematical Modelling and Computer Simulation (C-MMACS),
National Aeronautical Laboratory, Bangalore 560 017, India

and

ASHOK N. BHASKARWAR
Department of Chemical Engineering, Indian Institute of Technology, Hauz Khas,
New Delhi 110 016, India

(*Received* 30 *March* 1989 *and in final form* 6 *February* 1992)

Abstract—The problem of heat transfer to slurries is analysed theoretically. Three models of heat transfer are considered, namely, the thermal penetration model, the surface renewal model and the film model. Analytical expressions for the heat transfer coefficient and the enhancement factor are derived. It is found that the prediction of the enhancement in heat transfer is independent of the hydrodynamic model employed, in spite of the apparent differences in the expressions for the rates of heat transfer.

INTRODUCTION

DISPERSIONS of solid particles in fluid phases are encountered in a number of practical systems of interest to engineers. These include, among others, drying of wet granular materials or suspensions (such as paints, varnishes, lotions and foods), cooling of nuclear reactors (especially when twisted-tape inserts are used in the cooling systems) [1, 2], use of heterogeneous working fluids in Brayton space power generation cycle [3–5], process of extraction of heat from flue gas laden with alumina–silica catalyst in the petroleum industry [5] and fluidized-bed operations. Fluid flow visualization in the presence or absence of heat transfer is another scientific/engineering activity wherein finely divided glistening particles like polystyrene or aluminium powder are employed in suspension form [6, 7]. The performance of cooling systems depends upon the coolant properties, operating conditions and the design of equipment besides the materials of construction. A significant modification of the thermal transport properties may be achieved by heterogenizing the coolant fluid phase [8, 9]. A promissory example would be the use of a slurry of metal powder instead of a homogeneous fluid. The present paper is concerned with the theoretical investigation of heat transfer to slurries. Specifically, an attempt has been made to arrive at analytical expressions for the average rate of heat transfer and the enhancement factor. The theory has been developed along the guidelines of the analogous penetration and surface renewal models of mass transfer proposed by Higbie [10] and Danckwerts [11], respectively. A film model of heat transfer incorporating the theory of Brownian motion of particles has also been presented.

THE PHYSICAL SYSTEM

Figure 1 illustrates the physical system considered for the analysis of heat transfer to slurries. It also depicts the conceptual elements of the hydrodynamic models, namely, an 'element' of the penetration and the surface renewal models and a 'film' of the steady-state model. The one-dimensional coordinate system is represented by the positive half of the x-axis drawn perpendicular to the solid surface and with the origin placed at the surface.

THE THERMAL PENETRATION AND THE SURFACE RENEWAL MODELS

The following set of assumptions is chosen as a basis of the formulation of the mathematical model.

(1) The monodisperse particles in the suspension are spherical in shape, and very small in size, i.e. smaller than even the penetration scale of heat. They have a very high thermal conductivity. Further, it is assumed that $Nu = 2$ for the particles.

(2) The solid particles do not react with the liquid.

(3) The slurry can be regarded as a dilute system with low viscosity.

(4) The bulk of the slurry is well mixed and heat transfer to the bulk occurs via exchange of elements of slurry in the vicinity of the heated surface.

(5) The heat transfer problem is one-dimensional.

NOMENCLATURE

a_{cs}	cross-sectional area of a single particle [m^2]
a_p	surface area of a single particle [m^2]
A	surface area [m^2]
C_p	specific heat at constant pressure [$J\ kg^{-1}\ K^{-1}$]
d_p	diameter of a particle [m]
E_h	instantaneous enhancement in heat transfer
h	heat transfer coefficient [$W\ m^{-2}\ K^{-1}$]
i, j	summation indices
k	thermal conductivity [$W\ m^{-1}\ K^{-1}$]
K_1, K_2	constants defined in equations (4) and (5)
m	mass [kg]
n	summation index
n_p	number of particles per unit volume [m^{-3}]
N	Avogadro's number
Nu	Nusselt number
$\mathcal{PF}$	psdueo-function. Finite part of the diverging integral $\int_{-\infty}^{+\infty} f(t)\phi(t)\,dt = \langle f, \phi \rangle$, as defined by Hadamard
r	radius [m]
R	rate of heat transfer [$W\ m^{-2}$]
$\mathbb{R}$	universal gas constant [$J\ kmol^{-1}\ K^{-1}$]
s	speed of a particle [$m\ s^{-1}$]
T	absolute temperature [K]
t	time [s]
u	unit step function
x	spatial coordinate [m]
Z	average number of collisions of the particles with the interface.

Greek symbols

α	thermal diffusivity [$m^2\ s^{-1}$]
δ	film thickness [m]
Δ	impulse function
η	fluid viscosity [$kg\ m^{-1}\ s^{-1}$]
θ	nondimensional temperature
λ	fractional area offered by the particles
ξ	nondimensional spatial coordinate
ρ	density [$kg\ m^{-3}$]
τ	nondimensional time.

Subscripts

0	denotes initial conditions
l	liquid
ls	liquid to solid
p	particle
s	solid
sl	slurry
w	wall.

Superscripts

—	averaged over time
*	characteristic time scale.

The pertinent differential equation model appears as follows.

Liquid–side energy equation:

$$\frac{\partial T_l}{\partial t} = \alpha_l \frac{\partial^2 T_l}{\partial x^2} - \frac{h_{ls} n_p a_p}{\rho_l C_{pl}} (T_l - T_s). \tag{1}$$

Energy equation for a single particle:

$$m_p C_{ps} \frac{\partial T_s}{\partial t} = h_{ls} a_p (T_l - T_s). \tag{2}$$

Initial conditions:

$$\text{At } t = 0, \quad T_l = T_s = T_0. \tag{a}$$

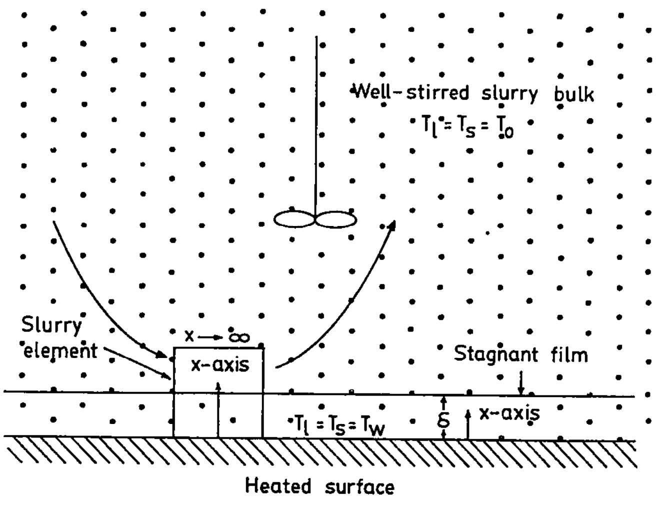


FIG. 1. Geometry and coordinate system.

Boundary conditions:

$$\text{At } t > 0, \quad x = 0, \quad T_\mathrm{l} = T_\mathrm{s} = T_\mathrm{w} \tag{b}$$

$$\text{At } t \geqslant 0, \quad x \to \infty, \quad T_\mathrm{l} = T_\mathrm{s} = T_0. \tag{c}$$

Nondimensionalization of equations (1) and (2) and conditions (a)–(c) is based on the following choice of variables:

$$\theta_\mathrm{l} = \frac{T_\mathrm{l} - T_0}{T_\mathrm{w} - T_0}, \quad \theta_\mathrm{s} = \frac{T_\mathrm{s} - T_0}{T_\mathrm{w} - T_0}, \quad \xi = x/d_\mathrm{p},$$

$$\tau = \alpha_\mathrm{l} t/d_\mathrm{p}^2. \tag{3}$$

The dimensionless equations resulting from equations (1)–(3) are

$$\frac{\partial \theta_\mathrm{l}}{\partial \tau} = \frac{\partial^2 \theta_\mathrm{l}}{\partial \xi^2} - K_1(\theta_\mathrm{l} - \theta_\mathrm{s}) \tag{4}$$

and

$$\frac{\partial \theta_\mathrm{s}}{\partial \tau} = K_2(\theta_\mathrm{l} - \theta_\mathrm{s}) \tag{5}$$

where

$$K_1 = \frac{h_\mathrm{ls} a_\mathrm{p} n_\mathrm{p} d_\mathrm{p}^2}{k_\mathrm{l}}$$

and

$$K_2 = \frac{h_\mathrm{ls} a_\mathrm{p} d_\mathrm{p}^2}{\alpha_\mathrm{l} m_\mathrm{p} C_\mathrm{ps}}.$$

Conditions (a)–(c) become

$$\text{at } \tau = 0, \quad \theta_\mathrm{l} = \theta_\mathrm{s} = 0.0 \tag{d}$$

$$\text{for } \tau > 0, \quad \xi = 0, \quad \theta_\mathrm{l} = \theta_\mathrm{s} = 1.0 \tag{e}$$

$$\tau \geqslant 0, \quad \xi \to \infty, \quad \theta_\mathrm{l} = \theta_\mathrm{s} = 0.0. \tag{f}$$

The system of partial differential equations (4) and (5), subject to conditions (d)–(f), has been solved analytically by the Laplace Transform technique [12] to obtain

$$\begin{aligned}\theta_\mathrm{l} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \Bigg[\xi^n \Bigg[\frac{\tau^{(-n/2)} u(\tau)}{\Gamma(1 - n/2)} & \\ + \sum_{j=1}^{\infty} {}^{n/2}C_j K_1^j \Bigg[\frac{\tau^{(j-n/2)} u(\tau)}{\Gamma(j+1-n/2)} & \\ + \sum_{i=1}^{\infty} {}^{-j}C_i K_2^i \frac{\tau^{(j-n/2-i)} u(\tau)}{\Gamma(j+1-n/2-i)} \Bigg]\Bigg]\Bigg] &\end{aligned}$$

$$\text{iff} \quad n < 2 \quad \text{and} \quad j > n/2 - 1 + i \tag{6}$$

$$\begin{aligned}\theta_\mathrm{l} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \Bigg[\xi^n \Bigg[\mathscr{P}\mathscr{F} \frac{\tau^{(-n/2)} u(\tau)}{\Gamma(1 - n/2)} & \\ + \sum_{j=1}^{\infty} {}^{n/2}C_j K_1^j \Bigg[\mathscr{P}\mathscr{F} \frac{\tau^{(j-n/2)} u(\tau)}{\Gamma(j+1-n/2)} & \\ + \sum_{i=1}^{\infty} {}^{-j}C_i K_2^i \mathscr{P}\mathscr{F} \frac{\tau^{(j-n/2-i)} u(\tau)}{\Gamma(j+1-n/2-i)} \Bigg]\Bigg]\Bigg] &\end{aligned}$$

$$\text{iff} \quad n > 2 \text{ (noninteger)} \quad \text{and} \quad j < n/2 - 1 + i \tag{7}$$

$$\begin{aligned}\theta_\mathrm{l} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \Bigg[\xi^n \Bigg[\Delta^{(n/2-1)} + \sum_{j=1}^{\infty} {}^{n/2}C_j K_1^j \Bigg[\Delta^{(j+1-n/2)} & \\ + \sum_{i=1}^{\infty} {}^{-j}C_i K_2^i \Delta^{(j+1-n/2-i)} \Bigg]\Bigg]\Bigg] &\end{aligned}$$

$$\text{iff} \quad n = 2, 4, 6, \ldots, \quad \text{and} \quad j = (n/2 - 1 + i) \tag{8}$$

and

$$\theta_\mathrm{s} = K_2 \, \mathrm{e}^{-K_2 \tau}. \tag{9}$$

Here, Δ is the impulse function, $u(\cdot)$ the unit step function, $\Gamma(\cdot)$ the Gamma function, and $\mathscr{P}\mathscr{F}$ the pseudo-function as defined by Hadamard [12]. A relation useful for the evaluation of the pseudo-function in equation (7) is

$$\mathscr{P}\mathscr{F} \frac{\tau^{(-n/2)} u(\tau)}{\Gamma(1 - n/2)} = -\mathscr{P}\mathscr{F} \frac{1}{\pi} \Gamma(n/2) \sin[\pi(n/2 - 1)] \frac{u(\tau)}{\tau^{n/2}}.$$

The instantaneous rate of heat transfer per unit area of a homogeneous liquid is

$$\begin{aligned} R = -k_\mathrm{l} \frac{\partial T_\mathrm{l}}{\partial x}\bigg|_{x=0} &= \rho_\mathrm{l} C_\mathrm{pl} (T_\mathrm{w} - T_0) \sqrt{\left(\frac{\alpha_\mathrm{l}}{\pi t}\right)} \\ &= \frac{k_\mathrm{l}(T_\mathrm{w} - T_0)}{d_\mathrm{p} \sqrt{(\pi \tau)}}. \end{aligned} \tag{10}$$

The corresponding rate for a slurry is

$$\begin{aligned} R_\mathrm{sl} &= \frac{-k_\mathrm{l}}{d_\mathrm{p}} (T_\mathrm{w} - T_0) \frac{\partial \theta_\mathrm{l}}{\partial \xi}\bigg|_{\xi=0} \\ &= \frac{k_\mathrm{l}(T_\mathrm{w} - T_0)}{d_\mathrm{p}} \Bigg[\frac{1}{\sqrt{(\pi\tau)}} + \sum_{j=1}^{\infty} {}^{1/2}C_j K_1^j \Bigg[\frac{\tau^{(j-1/2)} u(\tau)}{\Gamma(j+1/2)} \\ &\quad + \sum_{i=1}^{\infty} {}^{-j}C_i K_2^i \frac{\tau^{(j-i-1/2)} u(\tau)}{\Gamma(j-i+1/2)} \Bigg]\Bigg] \quad \text{iff } i < j + 1/2 \end{aligned} \tag{11}$$

$$\begin{aligned} R_\mathrm{sl} &= \frac{-k_\mathrm{l}}{d_\mathrm{p}} (T_\mathrm{w} - T_0) \frac{\partial \theta_\mathrm{l}}{\partial \xi}\bigg|_{\xi=0} \\ &= \frac{k_\mathrm{l}(T_\mathrm{w} - T_0)}{d_\mathrm{p}} \Bigg[\frac{1}{\sqrt{(\pi\tau)}} + \sum_{j=1}^{\infty} {}^{1/2}C_j K_1^j \Bigg[\mathscr{P}\mathscr{F} \frac{\tau^{(j-1/2)} u(\tau)}{\Gamma(j+1/2)} \\ &\quad + \sum_{i=1}^{\infty} {}^{-j}C_i K_2^i \mathscr{P}\mathscr{F} \frac{\tau^{(j-i-1/2)} u(\tau)}{\Gamma(j-i+1/2)} \Bigg]\Bigg] \end{aligned}$$

$$\text{iff} \quad i > j + 1/2. \tag{12}$$

The heat transfer enhancement factor is

$$\begin{aligned} E_\mathrm{h} = \frac{R_\mathrm{sl}}{R} = 1 + \sqrt{(\pi\tau)} \Bigg[\sum_{j=1}^{\infty} {}^{1/2}C_j K_1^j \Bigg[\frac{\tau^{(j-1/2)} u(\tau)}{\Gamma(j+1/2)} & \\ + \sum_{i=1}^{\infty} {}^{-j}C_i K_2^i \frac{\tau^{(j-i-1/2)} u(\tau)}{\Gamma(j-i+1/2)} \Bigg]\Bigg] & \end{aligned}$$

$$\text{iff} \quad i < j + 1/2 \tag{13}$$

and

$$E_{\mathrm{h}} = 1+\sqrt{(\pi\tau)}\left[\sum_{j=1}^{\infty} {}^{1/2}C_jK_1^j\left[\mathscr{P}\mathscr{F}\frac{\tau^{(j-1/2)}u(\tau)}{\Gamma(j+1/2)}\right.\right.$$
$$\left.\left.+\sum_{i=1}^{\infty} {}^{-j}C_iK_2^i\mathscr{P}\mathscr{F}\frac{\tau^{(j-i-1/2)}u(\tau)}{\Gamma(j-i+1/2)}\right]\right]$$
$$\text{iff}\quad i>j+1/2. \tag{14}$$

Special case. When the solid particles are absent, $K_1 = K_2 = 0$, and equations (13) and (14) imply that $E_{\mathrm{h}} = 1$, thus appropriately describing the limiting case of a homogeneous liquid. The average rate of heat transfer may be obtained from

$$\bar{R}_{\mathrm{sl}} = \frac{1}{\tau^*}\int_0^{\tau^*} R_{\mathrm{sl}}(\tau)\,\mathrm{d}\tau. \tag{15}$$

The average heat transfer coefficient may be defined as

$$\bar{R}_{\mathrm{sl}} = h_{\mathrm{sl}}(T_{\mathrm{w}}-T_0). \tag{16}$$

From equations (11), (12) and (15), we obtain

$$\bar{R}_{\mathrm{sl}} = \frac{k_1(T_{\mathrm{w}}-T_0)}{d_{\mathrm{p}}}\left[\frac{2}{\sqrt{(\pi\tau^*)}}\right.$$
$$+\sum_{j=1}^{\infty} {}^{1/2}C_jK_1^j\left[\frac{\tau^{*(j-1/2)}u(\tau^*)}{\Gamma(j+1/2)(j+1/2)}\right.$$
$$\left.\left.\left.+\sum_{i=1}^{\infty} {}^{-j}C_iK_2^i\left[\frac{\tau^{*(j-i-1/2)}u(\tau^*)}{\Gamma(j-i+1/2)(j-i+1/2)}\right]\right]\right]$$
$$\text{iff}\quad i<j+1/2 \tag{17}$$

and

$$\bar{R}_{\mathrm{sl}} = \frac{k_1(T_{\mathrm{w}}-T_0)}{d_{\mathrm{p}}}\left[\frac{2}{\sqrt{(\pi\tau^*)}}\right.$$
$$+\sum_{j=1}^{\infty} {}^{1/2}C_jK_1^j\left[\mathscr{P}\mathscr{F}\frac{\tau^{*(j-1/2)}u(\tau^*)}{\Gamma(j+1/2)(j+1/2)}\right.$$
$$\left.\left.\left.+\sum_{i=1}^{\infty} {}^{-j}C_iK_2^i\left[\mathscr{P}\mathscr{F}\frac{\tau^{*(j-i-1/2)}u(\tau^*)}{\Gamma(j-i+1/2)(j-i+1/2)}\right]\right]\right]$$
$$\text{iff}\quad i>j+1/2. \tag{18}$$

Expressions for the flux may also be written in terms of Bessel functions. For example, equations (17) and (18) may be written in the following form which is more compact:

$$\bar{R}_{\mathrm{sl}} = \frac{k_1(T_{\mathrm{w}}-T_0)}{d_{\mathrm{p}}}[\{(1+2B)\exp(-B)\}I_0(B)$$
$$+2BI_1(B)] \tag{19}$$

where

$$B = \tfrac{1}{2}(K_1+K_2)\tau^*.$$

Comparing equations (17) and (18) with equation (16), we obtain

$$\bar{h}_{\mathrm{sl}} = \frac{k_1}{d_{\mathrm{p}}}\left[\frac{2}{\sqrt{(\pi\tau^*)}}+\sum_{j=1}^{\infty} {}^{1/2}C_jK_1^j\left[\frac{\tau^{*(j-1/2)}u(\tau^*)}{\Gamma(j+1/2)(j+1/2)}\right.\right.$$
$$\left.\left.\left.+\sum_{i=1}^{\infty} {}^{-j}C_iK_2^i\left[\frac{\tau^{*(j-i-1/2)}u(\tau^*)}{\Gamma(j-i+1/2)(j-i+1/2)}\right]\right]\right]$$
$$\text{iff}\quad i<j+1/2 \tag{20}$$

and

$$\bar{h}_{\mathrm{sl}} = \frac{k_1}{d_{\mathrm{p}}}\left[\frac{2}{\sqrt{(\pi\tau^*)}}+\sum_{j=1}^{\infty} {}^{1/2}C_jK_1^j\left[\mathscr{P}\mathscr{F}\frac{\tau^{*(j-1/2)}u(\tau^*)}{\Gamma(j+1/2)(j+1/2)}\right.\right.$$
$$\left.\left.\left.+\sum_{i=1}^{\infty} {}^{-j}C_iK_2^i\left[\mathscr{P}\mathscr{F}\frac{\tau^{*(j-i-1/2)}u(\tau^*)}{\Gamma(j-i+1/2)(j-i+1/2)}\right]\right]\right]$$
$$\text{iff}\quad i>j+1/2. \tag{21}$$

Unlike the thermal penetration model, the surface renewal model presumes a distribution of lifetimes of elements from zero to infinity [11]. The relevant expressions based on the surface renewal model are obtained as follows:

$$\bar{R}_{\mathrm{sl}} = \bar{h}_{\mathrm{sl}}(T_{\mathrm{w}}-T_0) = \frac{k_1(T_{\mathrm{w}}-T_0)}{d_{\mathrm{p}}}$$
$$\times\sqrt{\left(p\left[1+\frac{K_1}{(p+K_2)}\right]\right)} \tag{22}$$

$$\bar{R} = \bar{h}(T_{\mathrm{w}}-T_0) = \frac{k_1(T_{\mathrm{w}}-T_0)}{d_{\mathrm{p}}}\sqrt{p} = \frac{k_1(T_{\mathrm{w}}-T_0)}{d_{\mathrm{p}}\sqrt{\tau^*}} \tag{23}$$

$$\bar{E}_{\mathrm{h}} = \sqrt{\left[1+\frac{K_1}{(p+K_2)}\right]} = \sqrt{\left[1+\frac{K_1k_1^2}{(\bar{h}^2d_{\mathrm{p}}^2+K_2k_1^2)}\right]} \tag{24}$$

and

$$\bar{h}_{\mathrm{sl}} = \bar{h}\sqrt{\left[1+\frac{K_1k_1^2}{(\bar{h}^2d_{\mathrm{p}}^2+K_2k_1^2)}\right]} \tag{25}$$

where

$$p = 1/\tau^* = \bar{h}^2d_{\mathrm{p}}^2/k_1^2 = \overline{Nu}^2.$$

Thus, the expressions included in the outermost brackets in equations (20) and (21) represent the average Nusselt number for the slurry-hot surface system according to the thermal penetration model. On the other hand, according to the surface renewal model

$$\overline{Nu}_{\mathrm{sl}} = \overline{Nu}\sqrt{\left[1+\frac{K_1k_1^2}{(\bar{h}^2d_{\mathrm{p}}^2+K_2k_1^2)}\right]}. \tag{26}$$

THE FILM MODEL

Heat transfer phenomena can also be modelled through a steady-state approximation. It may be postulated that heat transfer occurs simultaneously to the liquid and the particulate phase is idealised to be pseudo-homogeneous. The governing differential equations would appear as follows:

$$\frac{d^2T_l}{dx^2} - \frac{h_{ls}n_p a_p}{k_l}(T_l - T_s) = 0 \tag{27}$$

$$\frac{d^2T_s}{dx^2} + \frac{h_{ls}a_p}{m_p C_{ps}\alpha_s}(T_l - T_s) = 0 \tag{28}$$

$$\text{at } x = 0, \quad T_l = T_s = T_w \tag{g}$$

$$\text{at } x = \delta, \quad T_l = T_s = T_0. \tag{h}$$

The analytical solution of equations (27) and (28), subject to boundary conditions (g) and (h), is

$$\theta_l = \theta_s = 1 + (\theta_0 - 1)\frac{d_p\xi}{\delta}. \tag{29}$$

The rate of heat transfer is

$$\bar{R}_{sl} = -k_l A_l \left.\frac{dT_l}{dx}\right|_{x=0} - k_s A_s \left.\frac{dT_s}{dx}\right|_{x=0} \tag{30}$$

$$\bar{R}_{sl} = (k_l A_l + k_s A_s)\frac{(T_w - T_0)}{\delta}. \tag{31}$$

The enhancement factor is given by

$$\bar{E}_h = \frac{\bar{R}_{sl}}{\bar{R}} = 1 + \frac{k_s A_s}{k_l A_l}. \tag{32}$$

The ratio of the interfacial areas shared by the solid and the liquid may be estimated from the theory of Brownian motion [13, 14]. The average of the squares of a large number of displacements of a microscopically small particle is given by

$$\overline{\Omega^2} = \frac{\mathbb{R}Tt}{3\pi N\eta r_p}. \tag{33}$$

The average speed of a particle is therefore

$$\bar{s} = \sqrt{(\overline{\Omega^2})}/t = \sqrt{(\mathbb{R}T/3\pi N\eta r_p t)}. \tag{34}$$

The average number of collisions of particles with the interface is

$$Z = (n_p\bar{s})/2 \tag{35}$$

and the rate of fractional exposition of the particles to the interface is

$$\frac{d\lambda}{dt} = Za_{cs} = \tfrac{1}{2}n_p\sqrt{\left(\frac{\mathbb{R}T}{3\pi N\eta r_p t}\right)}\left[\frac{\pi d_p^2}{4}\right]. \tag{36}$$

The expression for the fractional area offered by the solid particles is

$$\lambda = \int_0^{t^*}\frac{d\lambda}{dt}dt = n_p\sqrt{(\mathbb{R}Tt^*\pi r_p^3/3N\eta)} \tag{37}$$

where

$$t^* = 12 \times \text{Relaxation time} = 8r_p^2\rho_s/3\eta.$$

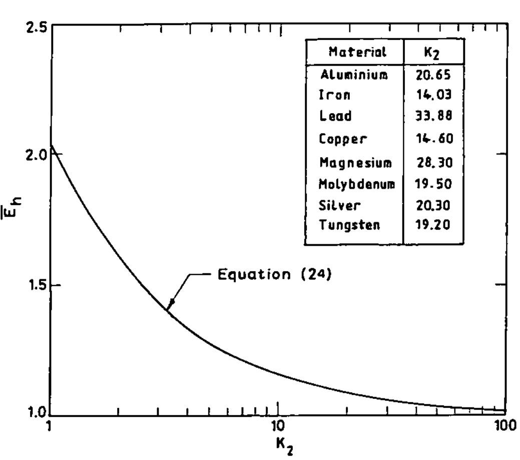
The relationship is in conformity with the assumption of Brownian motion as well as quasi-stationary particle motion [15]. It would appear that an analysis based on the Brownian motion is in conflict with the assumption of stationary particles made for describing heat transfer. It must however be remembered that the Brownian motion approach proposed here is itself only an approximation as heat transfer actually occurs under non-equilibrium conditions. The particle motion and heat transfer are thus regarded as uncoupled processes by presuming the particle motion under equilibrium conditions, corresponding to a reference temperature, to be valid under the actual conditions too.

RESULTS AND DISCUSSION

Figure 2 shows the dependence of the enhancement factor $\bar{E}_h$ on the material properties (equation (24)). It can be seen that the enhancement in heat transfer is highest for iron among the metals considered. Figure 3 presents two isometric views of a typical $\bar{E}_h(K_1, K_2)$ surface. The region of values of parameters K_1 and K_2 corresponding to significant enhancements in heat transfer rate can be clearly visualised from these three-dimensional plots. A sample calculation of the enhancement factor $\bar{E}_h$ from the surface renewal and the film models reveals a striking agreement. The values of parameters chosen for these calculations are given in Table 1. Alternatively, in terms of A_s/A_l we have, A_s/A_l (surface renewal model) $= 8.69 \times 10^{-4}$, a value which is calculated by equating equation (24) to equation (32), as compared to λ (which is nearly equal to A_s/A_l) $= 8.40 \times 10^{-4}$ for the film model. The ratio A_s/A_l as computed for the surface renewal model has no physical significance other than being an 'equivalent' to be compared to the quantity λ, given by equation (37). Further illustration of the agreement between these models is provided by Fig. 4 which is based on calculations for a broad range of magnitudes of the volume fraction of solids (Φ_s) in the slurry.

The above calculations bring out clearly the validity of using the theory of Brownian motion for estimating the fractional area occupied by the dispersed phase in the case of the film model, and this is a novel component of this analysis.

From the present analyses of linear systems, we find



Material	K_2
Aluminium	20.65
Iron	14.03
Lead	33.88
Copper	14.60
Magnesium	28.30
Molybdenum	19.50
Silver	20.30
Tungsten	19.20

FIG. 2. Dependence of the enhancement factor on the material properties of the particles.

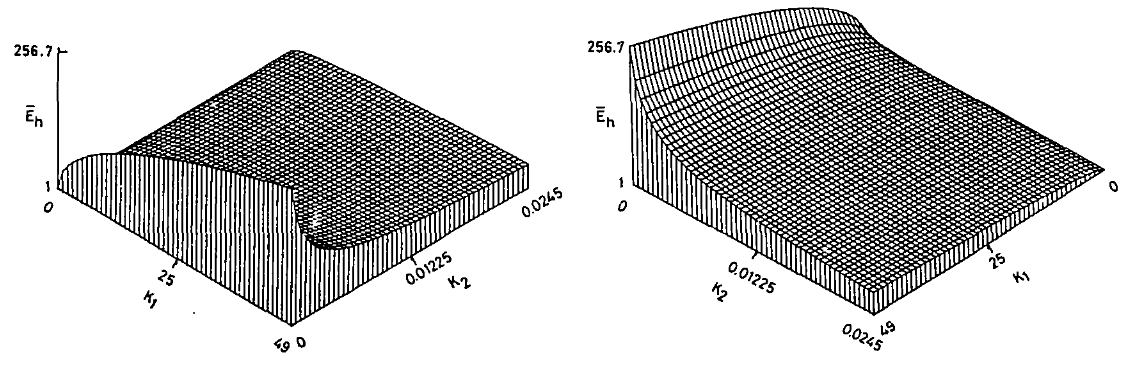


FIG. 3. Isometric view of the $\bar{E}_h(K_1, K_2)$ surface.

that the magnitudes of enhancement factors predicted by the different hydrodynamic models are essentially identical although the individual expressions for the rates of heat transfer appear to be quite different in form. Thus, these analyses are not only analogous to the conventional mass transfer models but also extend the ideas from a truly homogeneous situation to a pseudo-homogeneous one while still maintaining the mathematical linearity. The choice of a hydrodynamic model is not of much significance insofar as nearly the same magnitudes of enhancement factor are obtained.

The unsteady models are more general and acceptable while the film model is a workable approximation. The former interpret the microheterogeneity as sinks for the heat whereas the latter looks at heat transfer as parallel conduction through two types of continuous media even when one phase is dispersed. Both are some forms of pseudo-homogeneous models.

Finally, a few comments on the apparent contradictory effect of viscosity on the predictions of the models would be in order. According to the film model, if η is increased, λ would decrease by equation (37) and hence $\bar{E}_h$ would decrease. On the other hand, according to equation (24), which is obtained using the surface renewal theory, $\bar{h}$ would decrease if η is increased so that $\bar{E}_h$ will increase. This apparent contradiction can be resolved by observing that the present theory is valid only for dilute low viscosity systems. In the case of the surface renewal model, the factor d_p^2 multiplies $\bar{h}^2$ which is the only quantity

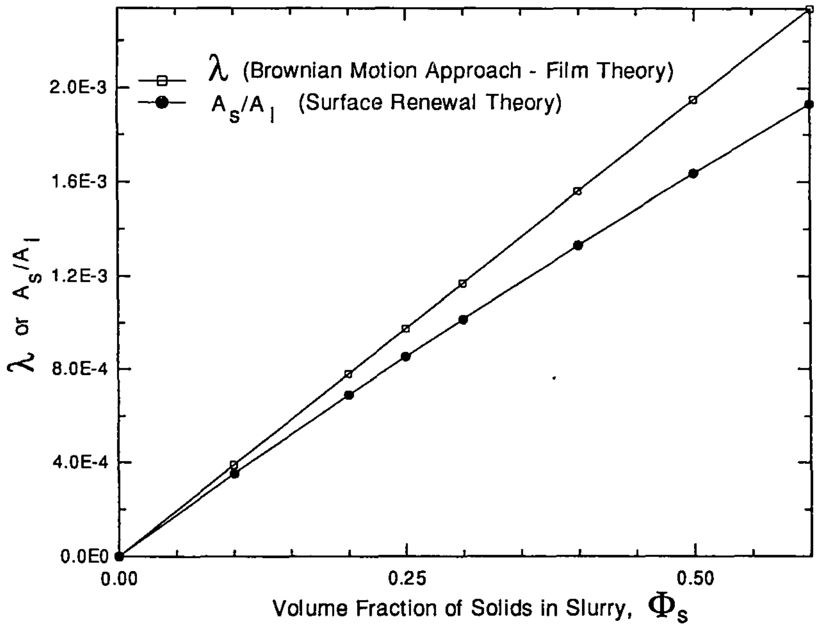


FIG. 4. Confirmation of equivalent predictions of heat transfer enhancement factor through the film and surface renewal theories for different solid loadings.

Table 1. Values of parameters

Parameter	Value
Thermal conductivity of liquid, k_l	0579 W m^{-1} K^{-1}
Thermal diffusivity of liquid, α_l	1.43×10^{-7} m^2 s^{-1}
Viscosity of liquid, η	1.0×10^{-3} kg m^{-1} s^{-1}
Heat transfer coefficient to the pure liquid, $\bar{h}$	15787 W m^{-2} K^{-1}
Particle diameter, d_p	1.0×10^{-6} m
Avogadro's number, N	6.023×10^{26} $kmol^{-1}$
Thermal conductivity of solid, k_s	73 W m^{-1} K^{-1}
Universal gas constant, $\mathbb{R}$	8.3144 kJ $kmol^{-1}$ K^{-1}
Specific heat of the solid at constant pressure, C_{ps}	0.452 kJ kg^{-1} K^{-1}

influenced by the liquid viscosity. However, d_p^2 is an extremely small number and hence the effect of viscosity is made very small. The film model represents a relatively stronger effect of viscosity, but it must be borne in mind that the application of the theory of Brownian motion to viscous systems would not be correct as the motion of particles would be slow and equation (36) would break down. For highly viscous systems, the ratio of areas (solid to liquid) would be largely governed by the fraction of cross-section occupied by the essentially immobile particles at the surface or in the bulk and the Brownian mechanism would be not only unwarranted but also inadequate. To sum up, the sensitivity of the film model to the viscosity of the liquid ceases to be contradictory to that of the surface renewal model because of the basic premise that these models are applicable only to the low viscosity dilute systems.

CONCLUSIONS

A novel approach based on the theory of Brownian motion has been proposed to estimate the fraction of heat transfer area exposed to the solid particles required in the parallel conduction film theory formulation.

The prediction of the enhancement in heat transfer due to monodisperse microheterogeneity is relatively independent of the choice of a hydrodynamic model. It would be interesting to see if the conclusions of this work logically extend to the situations involving polydisperse heat sinks.

Acknowledgements—The authors wish to thank the reviewer for his constructive criticism and insightful remarks.

REFERENCES

1. G. K. Rhode, D. M. Roberts, D. C. Schluderberg and E. E. Walsh, Gas-suspension coolants for power reactors, *Proc. Am. Pwr Conf.* **22**, 130–137 (1960).
2. D. C. Schluderberg, R. L. Whitelaw and R. W. Carlson, Gaseous suspensions—a new reactor coolant, *Nucleonics* **19**, 67–76 (1961).
3. R. Pfeffer, S. Rosetti and S. Lieblein, Analysis and correlation of heat transfer coefficient and friction factor data for dilute gas–solid suspensions, NASA TN D-3603 (1966).
4. R. Pfeffer, S. Rosetti and S. Lieblein, The use of a dilute gas–solid suspension as the working fluid in a single loop Brayton space power generation cycle, A.I.Ch.E. Paper-49c (1969).
5. C. A. Depew and T. J. Kramer, Heat transfer to flowing gas–solid mixtures. In *Advances in Heat Transfer*, Vol. 9, pp. 113–180. Academic Press, New York (1973).
6. T. Asanuma (Ed.), *Flow Visualisation Techniques in Japan*: *Flow Visualisation, Proc. Int. Symp. on Flow Visualisation*, 12–14 October, Tokyo, Japan, pp. 3–18 (1977).
7. W. J. Yang (Ed.), *Flow Visualisation III*: *Proc. Third Int. Symp. on Flow Visualisation*, 6–9 September, University of Michigan, Ann Arbor, Michigan, U.S.A. (1983).
8. A. E. Bergles, V. Nirmalan, G. H. Junkhan and R. L. Webb, *Bibliography on Augmentation of Convective Heat and Mass Transfer—II*, Heat Transfer Lab. Report HTL-31, ISU-ERI-Ames-84221, DE 84010848, Iowa State University, Ames, Iowa, U.S.A. (1983).
9. L. Farbar and M. J. Morley, Heat transfer to flowing gas–solid mixtures in a circular tube, *Ind. Engng Chem.* **49**, 1143–1150 (1957).
10. R. Higbie, Rate of absorption of a gas into a still liquid, *Trans. Am. Inst. Chem. Engrs* **35**, 365–389 (1935).
11. P. V. Danckwerts, Significance of liquid film coefficients in gas absorption, *Ind. Engng Chem.* **43**(6), 1460–1467 (1951).
12. G. Doetsch, *Introduction to the Theory and Application of the Laplace Transformation*. Springer, Berlin (1974).
13. E. D. Eastman and G. K. Rollefson, *Physical Chemistry*. McGraw-Hill, New York (1947).
14. H. S. Taylor (Ed.), *A Treatise on Physical Chemistry*. Van Nostrand, Princeton (1931).
15. N. A. Fuchs, *The Mechanics of Aerosols*. Pergamon Press, Oxford (1964).